Cosmological Models for the Perfect Fluid Universe in the Brans-Dicke Theory

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Dedicated to my parents

Abstract

Machian cosmological models are discussed in the Brans-Dicke theory for the homogeneous isotropic distribution of matter and radiation with the state equation $p=n\rho c^2$. The Machian condition dominates the evolution of matter in the universe, and is fixed to $GM/c^2a=k\pi$. The geometry of space is relevant to the coupling parameter of the scalar field. The universe is closed for the gravitation is attractive. The coupling parameter is also fixed to $\eta=-3$ cosmologically in the theory.

I. Introduction

General relativity was proposed as extension of special relativity to the arbitrary coordinate systems. However it is not merely extension, but includes the theory of gravitation necessarily. Einstein developed his theory inspired by Mach's ideas on relativity of space. Since then, it has been discussed to what extent Mach's ideas is contained in this theory, but still remains controversial and contentious. Brans and Dicke[1] *modified the Einstein theory in the* Machian point of view. There, however, also exist ambiguities concerning Mach's principle in their theory.

It seems that the problem lie down on cosmology rather than on the theory of gravitation itself. We live in this real universe. We can not discuss essentially real physical laws without reference to this universe. Physical phenomena are prescribed by the global distribution of matter in the universe and the gravitational interaction which lead to Machian properties. First we must determine a cosmological model of the real universe. The guiding principle is Mach.

In the Brans-Dicke theory[1] the gravitational interaction is described by the metric tensor of the Riemannian manifold and the scalar field on it. Because of complexity of the field equations, analytical comological solutions had not been found for an age. After a decade a simple solution for the closed universe was solved[2] on the assumption of a power law in the expansion parameter.

Later the same solution was rediscovered by means of Machian necessity[3,4]. In this closed cosmological model, in which the relation $GM/c^2a=\pi$ is always satisfied, the complete inertial frame dragging is realized when the rotating shell covers the whole universe[4]. This model also has some other interesting properties[5,6,7] in the Machian point of view, and is increasing its importance.

In this cosmological model, however, it is assumed that the pressure is zero (a dust model). It is sure that the interaction between galaxies is negligible in the present universe, and hence that the pressure is negligible, but in the era of the early universe radiation is superior to matter and the pressure of radiation is not negligible. If this Machian cosmological model is valid and a reflection of the real universe, it should be given the wider foundations.

In this paper, Machian cosmological models with an arbitrary state equation for the mixture of radiation and matter are discused.

II. Fundamental Equations

The field equations of the Brans-Dicke theory[1] which describe the gravitational interaction are written in our sign convention as

$$\begin{cases} R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\frac{8\pi}{c^4\phi}T_{\mu\nu}-\frac{\eta}{\phi^2}(\phi_{,\mu}\phi_{,\nu}-\frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}) \\ \qquad -\frac{1}{\phi}(\phi_{,\mu;\nu}-g_{\mu\nu}\Box\phi), & \text{(1a)} \\ \Box\phi=-\frac{8\pi}{(3+2\eta)c^4}T, & \text{(1b)} \end{cases}$$

where $T_{\mu\nu}$ is the energy-momentun tensor, which has for the perfect fluid the form

$$T_{\mu\nu}=-pg_{\mu\nu}-(\rho+p/c^2)u_\mu u_\nu, \tag{2}$$

in which ρ is the density in comoving coordinates, p is the pressure, and u^μ is the four-velocity $dx^\mu/d\tau$. The symbol $\Box$ denotes the generally covariant d'Alembertian $\Box\phi\equiv\phi_{,\mu;\mu}$, and the letter η is the coupling parameter between the scalar field ϕ and the contracted energy-momentum tensor T.

We assume the homogeneity and the isotropy of the universe, which seem to be satisfied globally in our universe. For the homogeneous and isotropic universe, the metric form of the Riemannian manifold will be written as

$$ds^2=-dt^2+a^2(t)[d\chi^2+\sigma^2(\chi)(d\theta^2+\sin^2\theta d\varphi^2)], \tag{3}$$

where

$$\sigma(\chi)\equiv\begin{cases}\sin\chi & \text{for } k=1 \quad \text{(closedspace)},\\ \chi & \text{for } k=0 \quad \text{(flat space)},\\ \sinh\chi & \text{for } k=-1 \quad \text{(open space)}.\end{cases}$$

We consider both matter and radiation as the contents of the universe. For the homogeneous isotropic distribution of matter and radation, the non-vanishing components of the enegy-momentum tensor (2) are

$$\begin{aligned} T_{00}&=-\rho c^2,\\ T_{11}&=-pa^2,\\ T_{22}&=-pa^2\sigma^2,\\ T_{33}&=-pa^2\sigma^2\sin^2\theta, \end{aligned} \tag{4}$$

and the contracted energy-momentum tensor is

$$T=\rho c^2-3p. \tag{5}$$

The density ρ is the sum of the density for matter and radiation: $\rho=\rho_m+\rho_r$. The pressure p is almost contributed from radiation.

The independent field equations of gravitation in the present problem are

$$2a\ddot{a}+\dot{a}^2+k=-\frac{\eta}{2}a^2\left(\frac{\dot{\phi}}{\phi}\right)^2+a\dot{a}\left(\frac{\dot{\phi}}{\phi}\right)-\frac{8\pi a^2}{(3+2\eta)c^4\phi}(2\eta p+\rho c^2), \tag{6a}$$

$$\frac{3}{a^2}(\dot{a}^2+k)=\frac{\ddot{\phi}}{\phi}+\frac{\eta}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{8\pi[2(1+\eta)\rho c^2+3p]}{(3+2\eta)c^4\phi}, \tag{6b}$$

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}=\frac{8\pi}{(3+2\eta)c^4}(\rho c^2-3p), \tag{6c}$$

where a dot denotes the derivative with respect to t. Unknown functions which we must solve are the expansion parameter $a(t)$, the scalar field $\phi(t)$, the density $\rho(t)$, and the pressure $p(t)$. It still more needs a state equation, which gives the relation between density and pressure, to determine unknown functions completely. We assume the state equation as

$$p = n\rho c^2, \qquad (0 \leqq n \leqq 1/3), \tag{7}$$

where n is restricted between 0 and 1/3. The upper bound of n is correspondent to the case which the universe is full of radiation.

III. Machian Cosmological Solutions

Matter and radiation yield the gravitational field in accordance with the field equations, and at the same time they must obey the conservation law of the energy-momentum $T^{\mu\nu}{}_{;\nu}=0$, which is derived from the field equations themselves. For the homogeneous isotropic distribution of matter and radiation, the conservation law is written down explicitly as

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0. \tag{8}$$

We can choose Eq. (8) as one of the independent field equations instead of Eq. (6a) for simplicity. Immediately by means of integration of Eq. (8) we obtain

$$\rho a^{\alpha} = K, \qquad \alpha \equiv 3(n+1), \tag{9}$$

where K is an arbitrary constant and α is restricted between 3 and 4.

Now there are some reasons why we expect and believe firmly the existence of a Machian solution in which the relation $GM/c^2a \sim 1$ is satisfied. And in fact such a solution exists.

Let us suppose

$$GM/c^2a = \text{const.}, \tag{10}$$

where

$$M(t) \equiv 2\pi^2 a^3 \rho \propto a^{3-\alpha}, \tag{11}$$

which means mass of the whole universe for the closed one. Equation (10) is more essential than the relation $a(t)\phi(t)=$const. in the Machian point of view in the present general problem. We can actually ascertain that a solution satisfying $a\phi=$const. does not exist generally. The gravitational "constant" G is proportional to the reciprocal of the scalar field $\phi(t)$, and so the relation (10) and the definition (11) lead to

$$\phi(t) = Da^{2-\alpha}, \tag{12}$$

where D is an arbitrary constant.

From Eqs. (9) and (12) we can easily obtain

$$\dot{\phi}/\phi = (2-\alpha)(\dot{a}/a) \tag{13}$$

and

$$\frac{\rho}{\phi} = \frac{K}{D} \cdot \frac{1}{a^2}. \tag{14}$$

First we eliminate $\ddot{\phi}$ in Eq. (6a) by means of Eq. (6c). After substitution of the state equation (7), Eq. (13), and Eq. (14) into the field equation (6b), we get

$$\left[\frac{\eta}{2}(\alpha-2)^2 + 3(\alpha-3)\right]\dot{a}^2(t) = 3k - \frac{8\pi}{c^2}\,\frac{K}{D}, \tag{15}$$

which means the time-derivative of the expansion parameter $\dot{a}(t)=$ const..

We assume the Big Bang as the beginning of the universe, and therefore we put the expansion parameter $a=0$ at $t=0$ as the initial condition to solve Eq. (15). Thus we find the time-dependence of the expansion parameter is

$$a(t) = A \cdot t, \tag{16}$$

where

$$A \equiv \left\{3k - \frac{8\pi}{c^2}\ \frac{K}{D} \Big/ \left[\frac{\eta}{2}(\alpha-2)^2 + 3(\alpha-3)\right]\right\}^{1/2}. \tag{17}$$

Using Eq. (16) we can easily find the time-dependence of $\dot{\phi}$, $\ddot{\phi}$, and ρ:

$$\dot{\phi}/\phi = (2-\alpha)t^{-1}, \tag{18}$$

$$\ddot{\phi}/\phi = (1-\alpha)(2-\alpha)t^{-2}, \tag{19}$$

$$\frac{\rho}{\phi} = \frac{K}{D} \cdot \frac{1}{A^2 t^2}. \tag{20}$$

Substituting these equations into another independent field equation (6c) we obtain

$$2-\alpha = \frac{8\pi}{(3+2\eta)c^2} \cdot \frac{K}{D} \cdot \frac{1}{A^2}. \tag{21}$$

By means of this equation the ratio K/D is determined automatically except $k=0$. For the flat space $k=0$, the ratio K/D becomes indefinite, but the value of the coupling parameter η is determined by Eq. (21). We get $\eta=-2$ for $k=0$.

In the Brans-Dicke theory the gravitational "constant" is described by the scalar field:

$$G \equiv \frac{4+2\eta}{3+2\eta} \cdot \phi^{-1}. \tag{22}$$

So the Machian assumption (10) is expressed in the present problem as

$$\frac{GM}{c^2 a} = \frac{2+\eta}{3+2\eta} \cdot \frac{4\pi^2 K}{c^2 D}. \tag{23}$$

Using the value of ratio K/D determined by Eq. (21) we get

$$\frac{GM}{c^2 a} = \frac{3k\pi(2+\eta)(\alpha-2)}{6-\eta(\alpha-2)(\alpha-6)}, \qquad (k \neq 0). \tag{24}$$

If the quantity $GM/c^2 a$ has this special value, there are no contradictions among Eqs. (6b), (6c), (7), and (8) which we must solve. Therefore the function (16) is a solution of the Brans-Dicke field equations for the homogenous isotropic universe filled with matter and radiation. The Machian solution which satisfies the relation $GM/c^2a=$const. exists only when that has a special value. The value of GM/c^2a is necessarily determined by the framework of the theory itself, and moreover its value is of the order of unity.

Similarly using the value of ratio K/D we can rewrite Ep. (15):

$$\dot{a}^2(t)=-6k/[6-\eta(\alpha-2)(\alpha-6)]. \tag{25}$$

In order that this equation has a physical meaning, that is, that the expansion parameter is real, the square of $a(t)$ is always positive for all α restricted between 3 and 4. Therefore

$$\eta<-2 \qquad \text{for} \quad k=1,$$
$$\eta>-3/2 \qquad \text{for} \quad k=-1.$$

By means of these conditions, for the closed space the Machian quantity GM/c^2a becomes positive and so the gravitational "constant" G is positive. For the open space the gravitional "constant" becomes negative. This means the gravitational force is attractive in the closed universe, repulsive in the open universe respectively in the Machian point of view.

After all the Brans-Dicke theory of gravitation has the following cosmological models:

$$ds^2=-dt^2+a^2(t)[d\chi^2+\sigma^2(\chi)(d\theta^2+\sin^2\theta d\varphi^2)],$$

$$\begin{cases} a(t)=\{6k/[\eta(\alpha-2)(\alpha-6)-6]\}^{1/2}\cdot t, \\ \phi(t)=-\dfrac{8\pi}{(\alpha-2)(3+2\eta)c^2}\cdot\rho(t)\cdot t^2, \\ \rho(t)=K\{[\eta(\alpha-2)(\alpha-6)-6]/6k\}^{\alpha/2}\cdot t^{-\alpha}, \\ p(t)=n\rho(t)c^2 \end{cases}$$

on the condition (24).

For the flat space $k=0$ we get the similar solution by putting

$k=0$ and $\eta=-2$ in Eq. (17). In this case the Machian quantity GM/c^2a cannot be determ!ned by the theory, but remains an arbitrary constant.

IV. Time-varying State Equation

In the previous section we discussed Machian cosmological models with the state equation (7) in which n is constant. However the ratio of density between matter and radiation varies in time as the universe expands, and still more the contents of the universe may change in time. Would a Machian cosmological solution exist in this more general situation, as expected ?

We begin with the state equation

$$p(t)=n(t)\rho(t)c^2, \qquad 0\leqq n(t)\leqq 1/3, \tag{26}$$

where $n(t)$ is no more constant. The independent field equations or the conservation law of the energy-momentum are Eqs. (6b), (6c), and (8). Unknown functions are the expansion parameter $a(t)$, the scalar field $\phi(t)$, the density $\rho(t)$, the pressure $p(t)$, and the state parameter $n(t)$. The number of unknown function is one more than that of independent equation. It is natural in the ordinary meaning. For the state of matter in the universe is not determined within the framework of the theory.

Nevertheless the Machian point of view dominates even the state of matter in the universe. Or it may be more moderate that we think the Machian condition is realized in the results of a real scenario of our universe. Anyway we investigate posibilities of the existence of a Machian cosmological model in the present situation. We add the Machian condition GM/c^2a=const. into the four independent equations as the fifth equation.

After integration of Eq. (8) for a variable $n(t)$ we get

$$\rho(t)=\rho_0 e^{-F(t)}, \tag{27}$$

where

$$\frac{dF}{dt}\equiv 3[n(t)+1]\frac{\dot{a}}{a}, \tag{28}$$

and ρ_0 is an arbitrary constant. So the Machian condition is equivalent to the equation

$$\phi(t)=Da^2(t)e^{-F(t)}, \tag{29}$$

where D is an arbitrary constant. The Machian condition is written explicitly as

$$\frac{GM}{c^2a}=\frac{2+\eta}{3+2\eta}\cdot\frac{4\pi^2\rho_0}{c^2D}\,. \tag{30}$$

From Eqs. (27) and (29) we calculate

$$\frac{\dot{\phi}}{\phi}=-(3n+1)\frac{\dot{a}}{a}, \tag{31}$$

$$\frac{\ddot{\phi}}{\phi}=(3n+1)(3n+2)\frac{\dot{a}^2}{a^2}-3\dot{n}\frac{\dot{a}}{a}-(3n+1)\frac{\ddot{a}}{a}, \tag{32}$$

$$\frac{\rho}{\phi}=\frac{\rho_0}{D}\cdot\frac{1}{a^2}. \tag{33}$$

By substitution of these equations into Eqs. (6b) and (6c) we obtain respectively

$$\begin{cases}\left[\frac{\eta}{2}(3n+1)^2+9n\right]\dot{a}^2(t)=3k-\frac{8\pi\rho_0}{c^2D}, & (34)\\ -(3n+1)a\ddot{a}+(3n+1)(3n-1)\dot{a}^2-3\dot{n}a\dot{a}+(3n-1)\frac{8\pi\rho_0}{(3+2\eta)c^2D}=0. & (35)\end{cases}$$

By elimination of $\ddot{a}$ in Eq. (35) by means of the differentiation of Eq. (34) we get

$$\frac{8\pi\rho_0}{(3+2\eta)c^2D}=\frac{3\dot{n}a\dot{a}-2k(3n+1)}{\eta(3n+1)(n-1)-2}\left(=\text{const.}\equiv N\right). \tag{36}$$

The left-hand side of Eq. (36) is constant, so the right-hand side must be invariant in time. Taking the real physical process into consideration we may put $\dot{n}\to 0$ and $n\to 0$ at $t\to +\infty$. We assume that $\dot{n}a\dot{a}$ converges to zero at plus infinity in time. This assumption will be checked by a solution later. Thus we get

$$N=2k/(2+\eta), \qquad (k\neq 0). \tag{37}$$

This means

$$\frac{GM}{c^2a}=k\pi, \qquad (k\neq 0). \tag{38}$$

Therefore we can rewrite Eqs. (34) and (35) respectively into

$$\left\{\begin{array}{ll} \left[\frac{\eta}{2}(3n+1)^2 9n\right]\dot{a}^2(t)=-\frac{k\eta}{2+\eta}, & (39) \\ 3(2+\eta)\dot{n}a\dot{a}=2kn[\eta(3n+1)+6]. & (40) \end{array}\right.$$

The time-dependence of the expansion parameter $a(t)$ and the state parameter $n(t)$ is determined simultaneously by means of these two equations. Thus the change of the state of matter in the universe is principally determined in the theory. We, however, suppose that we can not solve exactly these non-linear simultaneous differential equation.

Let us seek another way. Equation (39) gives the time-derivative of $a(t)$ for given $n(t)$, and almost determines the time-dependence of $a(t)$. Equation (40) describes the time-dependence of $n(t)$ for given $a(t)$. The state parameter $n(t)$ is restricted between 0 and 1/3, and maybe varies slowly in comparison with the expansion parameter $a(t)$. At least the middle bracket of the left-hand side of Eq. (39) changes its value very slowly in comparison with $a(t)$. Therefore in a short time we think the state parameter $n(t)$ is almost constant, and hence the derivative of the expansion parameter $\dot{a}(t)$ is constant.

For a constant n the solution of $a(t)$ is Eq. (16). Accordingly a solution of Eq. (39) may be expressed as

$$a(t)=A(n(t))\cdot t, \tag{41}$$

where

$$A\equiv\left\{[-(2+\eta)/k\eta]\left[\frac{\eta}{2}(3n+1)^2+9n\right]\right\}^{-1/2}$$

The coefficient $A(n)$ varies in time slowly as the state parameter $n(t)$ changes. We obtain

$$\dot{a}(t)=\frac{dA}{dn}\cdot\frac{dt}{dn}\cdot t+A. \tag{42}$$

If the condition

$$\frac{dA}{dn}\cdot\frac{dn}{dt}\cdot t\ll A(n) \tag{43}$$

is satisfied for all t, the solution (41) becomes good approximation enough.

After substitution of Eq. (41) and the derivative $\dot{a}(t)\sim A(n)$ into Eq. (40), we get

$$\frac{-3\eta t\dfrac{dn}{dt}}{\dfrac{\eta}{2}(3n+1)^2+9n}=2n[\eta(3n+1)+6]. \tag{44}$$

We can separate two variables, and integrate Eq. (44) easily. We find

$$\begin{aligned}-\frac{1}{3\eta}\mathrm{l_n}t=&\frac{2}{\eta(\eta+2)}\mathrm{l_n}n+\frac{1}{3(\eta+6)}\mathrm{l_n}|\eta(3n+1)+6|\\&-\frac{1}{6\eta}\mathrm{l_n}\left|\frac{\eta}{2}(3n+1)^2+9n\right|-\frac{3}{2\eta}G(n)+C,\end{aligned} \tag{45}$$

where

$$G(n)\equiv\begin{cases}\dfrac{2}{3\sqrt{-3(3+2\eta)}}\mathrm{arctg}\dfrac{2}{3\sqrt{-3(3+2\eta)}}\left[\dfrac{9}{2}\eta n+\dfrac{3}{2}(\eta+3)\right], & (k=1)\\[2ex]\dfrac{1}{3\sqrt{3(3+2\eta)}}\mathrm{l_n}\left|\dfrac{\dfrac{9}{2}\eta n+\dfrac{3}{2}(\eta+3)-\dfrac{2}{3\sqrt{3(3+2\eta)}}}{\dfrac{9}{2}\eta n+\dfrac{3}{2}(\eta+3)+\dfrac{2}{3\sqrt{3(3+2\eta)}}}\right|, & (k=-1)\end{cases}$$

and C is an arbitrary constant. This equation describes the time-dependence of the state parameter $n(t)$. The time-dependence of the expansion parameter $a(t)$ is derived by substitution of this function $n(t)$ into Eq. (41). Thus the Machian cosmological model has been gained.

In the last let us investigate the asymptotic behavior of the state parameter. At plus infinity ($t \to +\infty$) the state parameter converges to zero ($n \to 0$). The dominant part in Eq. (45) is

$$-\frac{1}{3\eta}\ln t \sim \frac{2}{\eta(\eta+6)}\ln n, \tag{46}$$

and hence we get

$$n(t) \sim t^{-(\eta+6)/6}. \tag{47}$$

In order that this $n(t)$ really converges to zero, the condition $\eta > -6$ is needed. From Eq. (40) this condition lead the result $\dot{n}(t) < 0$ for all t in the expanding universe $a(t) > 0$. The state parameter always decreases in the expanding universe. Using the asymptotic behavior of $n(t)$ we confirm $\dot{n}a\dot{a} \to 0$ for $t \to +\infty$, and the condition (43) for good approximation.

At $t \to 0$ the state parameter converges to 1/3. As the dominant part in Eq. (45) for the closed space $k=1$, the following function remains

$$-\frac{1}{3\eta}\ln t \sim \frac{1}{3(\eta+6)}\ln|\eta(3n+1)+6|. \tag{48}$$

The left-hand side must diverge as $n(t) \to 1/3$. Therefore we get

$$\eta = -3.$$

The coupling parameter of the scalar field is uniquely determined.

V. Discussions

Machian cosmological models, which satisfies the relation $GM/c^2a=$const., exist in the Brans-Dicke theory of gravitation not only in the case the state parameter is constant but also in the case it varies in time. The topology of the Riemannian manifold, in this type of cosmological models, is determined by the coupling parameter of the scalar field similarly in both cases:

$$\begin{cases} \eta<-2 & \text{for the closed space } k=1, \\ \eta=-2 & \text{for the flat space } k=0, \\ \eta>-3/2 & \text{for the open space } k=-1. \end{cases}$$

In the Friedmann cosmology the geometry of space is determined by the density of matter in the universe, which is an arbitrary parameter. However in the present cosmological models the density of matter (including radiation) or mass of the universe is not arbitrary, but it is determined selfconsistently by the theory, especially by the gravitational "constant".

It is remarkable that the value of the Machian relation is fixed within the framework of the theory even in general case that the state parameter varies in time as the universe expands, and still more that its value $k\pi$ is of the order of unity. This value is relevant to the change of the state of matter in the universe, and is directly derived from the final condition of the state parameter through Eq. (36).

The Machian relation $GM/c^2a=k\pi$ clearly indicate that the gravitational "constant" is positive for the closed universe $k=1$, negative for the open universe $k=-1$ respectively. For the flat universe $k=0$, the quantity GM/c^2a becomes indefinite. The gravitational force is attractive, and hence our universe must be closed in the Machian point of view.

The time-dependence of the expansion parameter $a(t)=At$ means that the universe expands linearly, or in another word galaxies expand with their own constant velocity. Therefore galaxies in the universe do not accelerate to each other. This means there exists a reference frame which does not accelerate to the distribution of matter in the universe. Acceleration to the distribution of matter has a unique meaning. We think the inertial force is induced by the acceleration to matter.

In general case the exact linearity of the expansion parameter is lost. However for a short time to describe local physical laws, we can regard that the universe expands uniformly for the state parameter varies slowly. Moreover influences of the deviation from linearity to the origin of the universe will cancel to each other owing to homogeneity and isotropy.

The Machian condition does not contradict to the four independent equations which describe the five unknown functions. Far from that it seems that the time-dependence of the state parameter prescribed by the Machian condition is a reasonable reflection of the real physical process of state changes. As the Machian condition is added from outside arbitrary by hand, so the state parameter may essentially varies nonsensically. But the solution is reasonable. We think that the Machian condition is meaningful in our universe.

We assumed in solving the state parameter initial and final conditions that $n\to 0$, $\dot{n}\to 0$ at $t\to +\infty$ and $n=1/3$ at $t=0$. We have found the final condition determines the value of the Machian relation. The initial condition request that the coupling parameter of the scalar field is $\eta=-3$. It has been told that the Brans-Dicke theory mars the beauty of general relativity by introducing the scalar field, that is, a beautiful theory should not have indefinite parameters as the coupling parameter which express the ratio of the scalar field and the metric tensor describing the gravitational field. Now we find that the coupling parameter is fixed to a definite value in the Machian point of view. We think that cosmological quantities are not given as arbitrary parameters, but are determined selfconsistently in feedback ways within the framfwork of the theory. Physical theories, especially the theory of gravitation should essencially be cosmological.

Brans and Dicke insisted[1] that the coupling parameter of the scalar field is positive and of the order of unity. We, however, think there are no reasons to be positive, on the contrary it should be negative in the Machian point of view[4]. A problem arrises from the magnitude of the coupling parameter, which is relevant observationally to the perihelion rotation of Mercury $\Delta_{B.D}=\Delta_{G.R}(4+3\eta)/(6+3\eta)$, the light deflection by the Sun, and so on. The magnitude of the coupling parameter ($\eta=-3$) obtained in the present investigation gives results which deviate from bounds of observational errors. Where should we seek the responsibility? We think that the Machian point of view is very fascinating and essential in cosmology.

The magnitude of the coupling parameter was derived from the initial condition for the state parameter. We need to investigate in detail the era of the early universe. Contents of the universe would probably become more various, and the state equation would be complex. The universe may be no more homogeneous and isotropic. The theory of gravitation itself may be ruined in the vicinity of the singurality.

Mach's principle does not prescribe a theory of gravitation, but it is a cosmological synthesis. Our universe functions organically as a united whole.

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